COHOMOLOGICAL CHARACTERIZATION OF UNIVERSAL BUNDLES ON GRASSMANNIANS OF LINES

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First

THE well-known criterion of Horrocks says that a vector bundle E on the complex projective space $\mathbb{P}^n$ splits (i.e. is isomorphic to a direct sum of line bundles $\mathcal{O}(l_i)$) if and only if it does not have intermediate cohomology. This result has been studied and generalized many times:

- In [4] G. Ottaviani made a cohomological characterization of when a vector bundle E is a direct sum of line bundles $\mathcal{O}(l_i)$ in Grassmannians and quadrics. In the particular case of the Grassmannian of $k-$planes in $\mathbb{P}^n$, the characterization includes the vanishing of cohomology of the tensor product of E with exterior powers of $\mathcal{Q}$, where $\mathcal{Q}$ is the rank$-(k+1)$ universal quotient bundle.
- In [1] E. Arrondo and B. Graña used Ottaviani's result to characterize the direct sums of vector bundles of the form $\mathcal{O}(l_i)$ and $\mathcal{Q}(m_j)$ for the particular case $\mathbb{G}(1,4)$.
- In [3] L. Costa and R. M. Miró-Roig gave a cohomological characterization of the sums of bundles of the form $\mathcal{Q}(l_i)$ in any Grassmannian.
- Finally, our starting point is the recent [2] where E. Arrondo and F. Malaspina give an improved version for the case $\mathbb{G}(1,n)$ of the characterization of sums of $\mathcal{O}(l_i)$ as follows:

Theorem. *A vector bundle E on the Grassmannian of lines $\mathbb{G}(1,n)$ splits if and only if the following conditions hold:*

i. $H^1_*(E\otimes\mathcal{Q}) = H^2_*(E\otimes S^2\mathcal{Q}) = H^3_*(E\otimes S^3\mathcal{Q}) = \ldots = H^{n-2}_*(E\otimes S^{n-2}\mathcal{Q}) = 0$

ii. $H^{n-1}_*(E\otimes S^{n-2}\mathcal{Q}) = H^{n}_*(E\otimes S^{n-3}\mathcal{Q}) = H^{n+1}_*(E\otimes S^{n-4}\mathcal{Q}) = \ldots = H^{2n-3}_*(E) = 0$

I have generalized this result by giving a cohomological characterization of direct sums of twists of $\mathcal{O},\mathcal{Q},S^2\mathcal{Q},\ldots,S^i\mathcal{Q}$ with $i\leq n-2$ using the ideas of [1].

My research

IN order to reach this generalization we have to do induction in the order of the symmetric power, where the case $i=0$ is the previous theorem of [2]. In each step of the induction we will have to remove one particular hypothesis and add a few more.

Theorem. *Let E be a vector bundle on the Grassmannian of lines $\mathbb{G}(1,n)$ and let $k\in\{0,1,\ldots,n-2\}$. Then E is a direct sum of twists of $\mathcal{O},\mathcal{Q},S^2\mathcal{Q},\ldots,S^k\mathcal{Q}$ if and only if the following conditions hold:*

i. $H^i_*(E\otimes S^i\mathcal{Q})=0 \quad i=1,2,\ldots,n-3,n-2$

ii. $H^i_*(E\otimes S^{i-(j+1)}\mathcal{Q})=0 \quad j=1,2,\ldots,k-1,k \quad k<n-2$
$i=j+1,j+2,\ldots,n-3,n-2$

iii. $H^i_*(E\otimes S^{j-i}\mathcal{Q})=0 \quad j=1,2,\ldots,k-1,k$
$i=1,2,\ldots,j-1,j$

iv. $H^i_*(E\otimes S^{(2n-3-j)-i}\mathcal{Q})=0 \quad j=0,1,2,\ldots,k-1,k$
$k<n-2 \quad i=n,n+1,\ldots,2n-j-4,2n-j-3$

v. $H^i_*(E\otimes S^{2n-j-2}\mathcal{Q})=0 \quad j=1,2,\ldots,k-1,k$
$i=2n-j-2,2n-j-1,\ldots,2n-4,2n-3$

vi. $H^{n-1}_*(E\otimes S^{n-k-2}\mathcal{Q})=0$

References

[1] E. Arrondo and B. Graña, *Vector bundles on $\mathbb{G}(1,4)$ without intermediate cohomology*, J. of Algebra 214 (1999), no.1, 128-142.

[2] E. Arrondo and F. Malaspina. *Cohomological characterization of vector bundles on Grassmannians of lines*. J. of Algebra 323 (2010),no.4, 1098-1106.

[3] L. Costa and R. M. Miró-Roig, *Cohomological characterization of vector bundles on multiprojective spaces*, J. of Algebra 294 (2005), no.1, 73-96, with a corrigendum in 319 (2008), no.3, 1336-1338.

[4] G. Ottaviani, *Some extension of Horrocks criterion to vector bundles on Grassmannians and quadrics*,Annali Mat. Pura Appl. (IV) 155 (1989), 317-341.